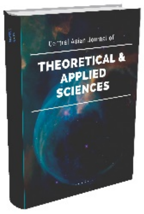

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Use of Elements of Predicate Algebra in Solving Proof Problems

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Annotation *This article explains the importance of using the laws of mathematical logic, its symbolism, and equally powerful formulas in the study of mathematics. The theorems presented in the article can be used to teach students the properties of a set of integers and the application of predicate algebra to examples and problem solving.*

Keywords: *Mathematical logic, predicate algebra, proof of theorem, equally strong formulas, whole numbers, even numbers, odd numbers.*

Mathematics is studied and developed on the basis of the laws of mathematical logic. However, mathematical logic is not taught as a separate subject in secondary schools. Although elements of mathematical logic are included in mathematics textbooks, their applications are not sufficiently covered. As a result, students face many difficulties in in-depth study of the theoretical foundations of mathematics, in solving examples and problems, especially in solving proof problems. With this in mind, in this article we will consider the application of predicate algebra to solving proof problems. In proving the theorems we use the following equally powerful formulas of predicate algebra.

$$P(x)\wedge(S(x)\vee Q(x))\equiv P(x)\wedge S(x)\vee P(x)\wedge Q(x) \quad (1)$$

$$P(x)\vee S(x)\wedge Q(x)\equiv(P(x)\vee S(x))\wedge(P(x)\vee Q(x)) \quad (2)$$

$$\overline{(P(x)\wedge S(x))}\equiv\overline{P}(x)\vee\overline{S}(x) \quad (3)$$

$$\overline{(P(x)\vee S(x))}\equiv\overline{P}(x)\wedge\overline{S}(x) \quad (4)$$

$$P(x)\Rightarrow S(x)\equiv\overline{P}(x)\vee S(x) \quad (5)$$

$$P(x)\Rightarrow S(x)\equiv\overline{S}(x)\Rightarrow\overline{P}(x) \quad (6)$$

$$P(x)\Leftrightarrow S(x)\equiv P(x)\wedge S(x)\vee\overline{P}(x)\wedge\overline{S}(x) \quad (7)$$

We denote the set of integers by the letter Z, the predicate "x-even number" by P (x), and the predicate "x-odd number" by S (x).

Theorem 1. $\forall(x,y\in Z)(P(x+y)\Rightarrow(P(x)\wedge P(y)\vee\overline{P}(x)\wedge\overline{P}(y))$ (8)

Proof. Conversely, we use the method of assumption (equation (6)).

$$\overline{(P(x)\wedge P(y)\vee\overline{P}(x)\wedge\overline{P}(y))}\equiv\overline{(P(x)\wedge P(y))}\wedge\overline{(\overline{P}(x)\wedge\overline{P}(y))}\equiv$$

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$\equiv(\overline{P}(x)\vee\overline{P}(y))\wedge(P(x)\vee P(y))\equiv\overline{P}(x)\wedge P(x)\vee\overline{P}(x)\wedge P(y)\vee$

$\vee\overline{P}(y)\wedge P(x)\vee\overline{P}(y)\wedge P(y)\equiv 0\vee\overline{P}(x)\wedge P(y)\vee\overline{P}(y)\wedge P(x)\vee 0\equiv$

$\equiv\overline{P}(x)\wedge P(y)\vee\overline{P}(y)\wedge P(x)\Rightarrow\overline{P}(x+y)\vee\overline{P}(x+y)\Rightarrow\overline{P}(x+y)$.

Hence, $\forall(x,y\in N)(\overline{(P(x)\wedge P(y)\vee\overline{P}(x)\wedge\overline{P}(y))}\Rightarrow\overline{P}(x+y)$. (9)

the confirmation is a true consideration. From this and (9) ≡ (8) it follows that the affirmation (8) is also true. The theorem is proved.

Theorem 2. $\forall(x,y\in Z)(S(x+y)\Rightarrow S(x)\wedge\overline{S}(y)\vee\overline{S}(x)\wedge S(y))$.(10)

Proof. Instead, we use the hypothesis method, which means that instead of stating that formula 10 is true, the following equation,

$\forall(x,y\in Z)(\overline{(S(x)\wedge\overline{S}(y)\vee\overline{S}(x)\wedge S(y))}\Rightarrow\overline{S}(x+y))$ (11)

Here is the formula:

$\overline{(S(x)\wedge\overline{S}(y)\vee\overline{S}(x)\wedge S(y))}\equiv\overline{(S(x)\wedge\overline{S}(y))}\wedge\overline{(\overline{S}(x)\wedge S(y))}\equiv$

$(\overline{S}(x)\vee S(y))\wedge(S(x)\vee\overline{S}(y))\equiv\overline{S}(x)\wedge S(x)\vee\overline{S}(x)\wedge\overline{S}(y)\vee S(y)\wedge S(x)\vee$

$\vee S(y)\wedge\overline{S}(y)\equiv 0\vee\overline{S}(x)\wedge\overline{S}(y)\vee S(x)\wedge S(y)\vee 0\equiv$

$\equiv\overline{S}(x)\wedge\overline{S}(y)\vee S(x)\wedge S(y)\Rightarrow\overline{S}(x+y)\vee\overline{S}(x+y)\Rightarrow\overline{S}(x+y)$. It turns out that formula (11) is a realistic consideration. The theorem is proved.

The following theorems are proved to be similar to Theorems 1 and 2.

Theorem 3. $(\forall x,y\in Z)(P(x-y)\Rightarrow P(x)\wedge P(y)\vee\overline{P}(x)\wedge\overline{P}(y))$

Theorem4. $(\forall x,y\in Z)(P(x\cdot y)\Rightarrow P(x)\wedge P(y)\vee P(x)\wedge\overline{P}(y)\vee\overline{P}(x)\wedge P(y))$

Theorem 5. $(\forall x,y\in Z)(P(x-y)\Rightarrow P(x+y))$

Theorem 6. $(\forall x,y\in Z)(S(x-y)\Rightarrow S(x)\ \overline{S}(y)\vee\overline{S}(x)\wedge S(y))$

Theorem 7. $(\forall x,y\in Z)(S(x-y)\Rightarrow S(x+y))$

Theorem 8. $(\forall x,y\in Z)(S(x\cdot y)\Rightarrow S(x)\wedge S(y))$

Theorem 9. $(\forall x,y\in Z)(P(x^2-y^2)\Rightarrow P(x)\wedge P(y)\vee\overline{P}(x)\wedge\overline{P}(y))$

Proof. We use the inverse assumption method.

$\overline{(P(x)\wedge P(y)\vee\overline{P}(x)\wedge\overline{P}(y))}\equiv\overline{(P(x)\wedge P(y))}\wedge\overline{(\overline{P}(x)\wedge\overline{P}(y))}\equiv$

$\equiv(\overline{P}(x)\vee\overline{P}(y))\wedge(P(x)\vee P(y)\equiv\overline{P}(x)\wedge P(x)\vee\overline{P}(x)\wedge P(y)\vee$

$\vee\overline{P}(y)\wedge P(x)\vee\overline{P}(y)\wedge P(y)\equiv 0\vee\overline{P}(x)\wedge P(y)\vee P(x)\wedge\overline{P}(y)\vee 0\equiv$

$\equiv\overline{P}(x)\wedge P(y)\vee P(x)\wedge\overline{P}(y)\Rightarrow\overline{P}(x-y)\wedge\overline{P}(x+y)\vee\overline{P}(x-y)\wedge\overline{P}(x+y)$

$\Rightarrow\overline{P}((x-y)\cdot(x+y))\Rightarrow\overline{P}(x^2-y^2)$. The theorem is proved.

Theorem 10. $(\forall x,y\in Z)(S(x^2-y^2)\Rightarrow S(x)\wedge\overline{S}(y)\vee\overline{S}(x)\wedge S(y))$

Isbot. $\overline{(S(x)\wedge\overline{S}(y)\vee\overline{S}(x)\wedge S(y))}\equiv(\overline{S}(x)\vee S(y))\wedge(S(x)\vee\overline{S}(y))\equiv$

$\equiv\overline{S}(x)\wedge\overline{S}(y)\vee S(x)\wedge S(y)\Rightarrow\overline{S}(x-y)\wedge\overline{S}(x+y)\vee\overline{S}(x-y)\wedge\overline{S}(x+y)\Rightarrow$

$\Rightarrow\overline{S}((x-y)\cdot(x+y))\vee\overline{S}((x-y)\cdot(x+y))\equiv\overline{S}(x^2-y^2)$. The theorem is proved.

Theorem 11. $(\forall x,y\in Z)(P(x)\wedge P(y)\vee\overline{P}(x)\wedge\overline{P}(y)\Rightarrow P(x^2-y^2))$

Proof. $P(x)\wedge P(y)\vee\overline{P}(x)\wedge\overline{P}(y)\Rightarrow P(x-y)\wedge P(x+y)\vee P(x-y)\wedge$

$\wedge P(x+y)\equiv P(x-y)\wedge P(x+y)\Rightarrow P((x-y)\cdot(x+y))\Rightarrow$

$\Rightarrow P(x^2-y^2)$. The theorem is proved.

Let R be a set of real numbers.

Theorem 12. $(\forall x\in R)\ (x\in[0;1]\Rightarrow x^2\leqslant x)$

Proof. We use the reverse assumption method.

$\overline{(x^2\leqslant x)}\Rightarrow x^2>x\Rightarrow x^2-x>0\Rightarrow x(x-1)>0\Rightarrow$

$\Rightarrow(x>0)\wedge(x-1>0)\vee(x<0)\wedge(x-1<0)\Rightarrow$

$\Rightarrow(x>0)\wedge(x>1)\vee(x<0)\wedge(x<1)\Rightarrow(x>1)\vee(x<0)\Rightarrow$

$\Rightarrow\overline{(x\in[0;1])}$. The theorem is proved.

Theorem 13. $(\forall x\in R)\ \ (x\in[0;1]\Rightarrow x^3\leqslant x^2)$

Proof. We use the reverse assumption method.

$\overline{(x^3\leqslant x^2)}\Rightarrow x^3>x^2\Rightarrow x^2(x-1)>0\Rightarrow(x^2>0)\wedge(x-1>0)\vee$

$\vee(x^2<0)\wedge(x-1<0)\Rightarrow(x^2>0)\wedge(x>1)\vee(x^2<0)\wedge(x<1)\Rightarrow$

$\Rightarrow(x\neq0)\wedge(x>1)\vee0\wedge(x<1)\Rightarrow(x>1)\vee0\Rightarrow x>1\Rightarrow$

$\Rightarrow x\in\overline{([0;1])}$. The theorem is proved.

Theorem 14. $(\forall x\in R)\ \ (x^2\leqslant x\Rightarrow x^3\leqslant x^2)$

Proof. We use the reverse assumption method.

$\overline{(x^3\leqslant x^2)}\Rightarrow x^3>x^2\Rightarrow x^2(x-1)>0\Rightarrow(x^2>0)\wedge(x-1>0)\vee$

$\vee(x^2<0)\wedge(x-1<0)\Rightarrow(x\neq0)\wedge(x>1)\vee0\wedge(x<1)\Rightarrow$

$\Rightarrow(x>1)\vee0\Rightarrow x>1\Rightarrow x^2>x\Rightarrow\overline{(x^2\leqslant x)}$. The theorem is proved.

The following theorems can also be proved using the inverse hypothesis method.

Theorem 15. $(\forall x\in R)\ \ (x\in[0;1]\Rightarrow x^3\leqslant x)$.

Theorem 16. $(\forall x\in R)\ \ ((x^2\leqslant x)\Rightarrow(x^3\leqslant x))$.

Theorem 17. $(\forall x\in R)\ \ ((|2x-1|\leqslant1)\Rightarrow(x\leqslant\sqrt{x}))$.

Theorem 18. $(\forall x\in R)\ \ (x^2\leqslant x\Rightarrow x\leqslant\sqrt{x})$.

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Theorem 19. $(\forall x \in R)$ $((x \leqslant \sqrt{x}) \Rightarrow x^2 \leqslant \sqrt{x}))$.

Theorem 20. $(\forall x \in R)$ $((\sqrt{x} \geqslant x) \Rightarrow (x^3 \leqslant x^2))$.

Theorem 21. $(\forall x,y \in R)$ $(|x|+|y| \leqslant 2 \Rightarrow x^2+y^2 \leqslant 4)$.

Theorem 22. $(\forall x,y \in R)$ $(|x|+|y| \leqslant 3 \Rightarrow x^2+y^2 \leqslant 9)$.

Theorem 23. $(\forall x,y \in R)$ $(|x|+|y| \leqslant 1 \Rightarrow x^2+y^2 \leqslant 1)$.

The above theorems can be used to develop students' problem-solving skills.

References

1. Yunusov A.S. Elements of the theory of mathematical logic and algorithms. T .: “Yangi asr avlodi”, 2006.
2. Kochkarov A. Ismailov Sh. Logical issues. T .: “Yangi asr avlodi”, 2008.
3. Igoshin V.I. Practical problem book on mathematical logic. M .: "Education", 1986.
4. Mendelssohn E. Introduction to mathematical logic. M .: "Science", 1984.

